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Spatial resolution of polarization imaging by partially coherent and partially polarized light

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ABSTRACT

The spatial resolution for polarization imaging system under illumination of any light source with arbitrary polarization and coherence has been investigated. Based on new factors referred to as the degree of polarization coherence, the resolution for polarization imaging has been studied as possible separations of two points in Stokes parameters with criteria charactering the accuracy of polarimetric images. Under the same illumination condition, these two closely spaced points resolvable for certain Stokes parameter will become unresolved for other Stokes parameters.

Keywords: Two-point resolution, polarization imaging, partially coherent and partially polarizedillumination

1. INTRODUCTION

In physics or especially in optics, spatial resolution describes the ability to distinguish small details of an object or to separate points of an object that are located at a small distance. As a major determinant, spatial resolution has played a critical role in any image-forming device, such as an optical or radio telescope, a microscope, a camera, and an eye [1]. Since the seminal work by Lord Rayleigh, a lot of efforts have been made to investigate the effects of the numerical aperture and various illumination conditions with partial coherence on imaging system's resolution [2-7].

On the other hand, the polarization imaging as an advanced sensing technique has attracted more and more interests due to its unique ability to detect the polarization information of objects, which will be beneficial to many applications, such as biomedical diagnostics and target detection, recognition and identification [8-9]. Although various principles and applications of polarization imaging have been developed in the last decades, few investigations have been made to the resolving power for a polarization imaging system.

In this letter, we study the two-point resolution for polarization imaging under illumination of a light source with arbitrary polarization and coherence. After an introduction of a set of normalized factors referred to as the degree of the generalized Stokes parameters, the spatial resolution in polarization imaging system has been revisited as the possible separation of the two points in Stokes parameters as measurable quantities. Criteria of spatial resolution for polarization imaging have been proposed to characterize how accurate the separation of two points can be achieved in Stokes parameters under varying conditions of coherence and polarization of the illumination.

2. DISTRIBUTION OF STOKES IMAGES

The object under consideration consists of two-closely separated points under illumination of a partially coherent and partially polarized light source (S). A combination of a polarizer (p) and a quarter wave plate (QWP) has been made use of for polarimetry with detection of Stokes parameters in the image plane (Viewing Screen). Under such illumination, the generalized Stokes parameters at object plan can be then approximated by:

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$$S_l^o(\xi_1,\eta_1;\xi_2,\eta_2) = S_l^s(\xi_1,\eta_1;\xi_2,\eta_2)[\delta(\xi_1-\Delta,\eta_1)+\delta(\xi_1+\Delta,\eta_1)]\times[\delta(\xi_2-\Delta,\eta_2)+\delta(\xi_2+\Delta,\eta_2)],\ \ l=0\sim 3 \qquad (1)$$

where (ξ,η) is the coordinate of position vector $\mathbf{r}$ in the object space, 2Δ represents the separation of the two transmitting points. $S_l(\xi_1,\eta_1;\xi_2,\eta_2), (l=0\sim 3)$ with their superscripts "o" and "s" indicating object and source illumination, respectively, are the generalized Stokes parameters defined by [10,11]:

$$S_0(\mathbf{r}_1;\mathbf{r}_2) = <\tilde{E}_x^*(\mathbf{r}_1)\tilde{E}_x(\mathbf{r}_2)> + <\tilde{E}_y^*(\mathbf{r}_1)\tilde{E}_y(\mathbf{r}_2)>, \quad (2.1)$$

$$S_1(\mathbf{r}_1;\mathbf{r}_2) = <\tilde{E}_x^*(\mathbf{r}_1)\tilde{E}_x(\mathbf{r}_2)> - <\tilde{E}_y^*(\mathbf{r}_1)\tilde{E}_y(\mathbf{r}_2)>, \quad (2.2)$$

$$S_2(\mathbf{r}_1;\mathbf{r}_2) = <\tilde{E}_x^*(\mathbf{r}_1)\tilde{E}_y(\mathbf{r}_2)> + <\tilde{E}_y^*(\mathbf{r}_1)\tilde{E}_x(\mathbf{r}_2)>, \quad (2.3)$$

$$S_3(\mathbf{r}_1;\mathbf{r}_2) = i[<\tilde{E}_y^*(\mathbf{r}_1)\tilde{E}_x(\mathbf{r}_2)> - <\tilde{E}_x^*(\mathbf{r}_1)\tilde{E}_y(\mathbf{r}_2)>], \quad (2.4)$$

where $<\cdots>$ indicates ensemble average, $\tilde{E}_x$ and $\tilde{E}_y$ are the components of the complex electric field vector represented by analytic signals. Under paraxial approximation, the random electromagnetic beam is assumed to propagate in a birefringence-free space–invariant system without high NA and the polarization image of the object expressed in the Stokes parameters can be found from the imaging equation:

$$S_l^i(x,y) = \iiint\int_{-\infty}^{\infty} h(x-\xi_1,y-\eta_1)h^*(x-\xi_2,y-\eta_2)S_l^o(\xi_1,\eta_1;\xi_2,\eta_2)d\xi_1 d\eta_1 d\xi_2 d\eta_2, \qquad (3)$$

where the superscript "i" indicates image space with (x, y) being it coordinate, and $h(x,y;\xi,\eta)$ is the complex-amplitude impulse response of the imaging system. After substitution of Eq.(1) into Eq.(3) we obtain the ordinary Stokes parameters in the image. Those are:

$$S_l^i(x,y) = S_l^s(\Delta,0)\left|h(x-\Delta,y)\right|^2 + S_l^s(-\Delta,0)\left|h(x+\Delta,y)\right|^2 + 2\,Re\left[S_l^s(\Delta,0;-\Delta,0)h(x-\Delta,y)h^*(x+\Delta,y)\right], \qquad (4)$$

where $Re\{\cdots\}$ and $Im\{\cdots\}$ denote the real and imaginary parts, respectively. In arriving at these results, we have made use of the relations, $S_l(\xi_1,\eta_1;\xi_2,\eta_2) = S_l^*(\xi_2,\eta_2;\xi_1,\eta_1)$ and $S_l(\xi_1,\eta_1;\xi_1,\eta_1) = S_l(\xi_1,\eta_1)$ from the definitions of the ordinary and generalized Stokes parameter and from the assumption of stationarity. It's the assumption of a birefringence-free imaging system that make the performance of polarization imaging similar for all S_l $(l=0\sim 3)$ with simple mathematical analysis, although the possibility of multi-modal imaging from the different choice of Stokes parameters has been restricted.

Equation (4) can be simplified further after introduction of several normalizations [12]. Similar to the degree of coherence, it is convenient to normalize the generalized Stokes parameters by setting:

$$\mu_{S_l}(\mathbf{r}_1;\mathbf{r}_2) = S_l(\mathbf{r}_1;\mathbf{r}_2)\big/\sqrt{2A(\mathbf{r}_1)A(\mathbf{r}_2)}, \qquad (5)$$

where μ_{S_l} may be referred to as the complex degree of Stokes polarization coherence elucidating the polarization and coherence properties of a stochastic electric fields, and $A_c(\mathbf{r}) = \sqrt{\sum_l S_l^2(\mathbf{r})/2}$ is the amplitude of the coherence tensor wave. From the definition in Eq. (5), it's not hard to find the possible ranges for μ_{S_l}, i.e. $0 \le |\mu_{S_l}| \le 1/\sqrt{2}$. When $\mathbf{r}_1$=$\mathbf{r}_2$=$\mathbf{r}$, the generalized Stokes parameters become the ordinary Stokes parameter and the corresponding μ_{S_l}. It is interesting to note that $\tilde{\mu}_{S_0}$ is identical to the degree of coherence of stochastic electromagnetic fields introduced by Wolf [12]. The ordinary Stokes parameters $S_l(\xi,\eta)$ can also be normalized by the total intensity S_0, i.e. $\hat{s}_l = S_l/(\sqrt{2}A)$ $(l=1,2,3)$. From their definitions above, it's no hard to find their relationship $\tilde{\mu}_{S_l}(\mathbf{r};\mathbf{r})$=$s_l$.

Since we are interested in the two-point resolution for polarization imaging under the illumination of the partially coherent and partially polarized light, we will focus our attention to the distribution of the image along the line passing through the centers of the two points by putting $y=0$ in the discussion to follow. Due to the fact that two points are closely spaced, these two points have been assumed to have equal illumination intensity and the same polarization properties, i.e. $A_c(\Delta,0)=A_c(-\Delta,0)=A_c$ and $\hat{s}_l(\Delta,0)=\hat{s}_l(-\Delta,0)=\hat{s}_l$ for mathematical simplicity without loss of generality. After some straightforward algebra, the Stokes parameters in image plane take the forms

$$S_l^i(x,0)=\sqrt{2}A_c\left\{\hat{s}_l\left[|h(x-\Delta,0)|^2+|h(x+\Delta,0)|^2\right]+2\,Re[\mu_{S_l}(\Delta,0;-\Delta,0)h(x-\Delta,0)h^*(x+\Delta,0)]\right\}. \tag{6}$$

For a diffraction limited system with a circular exit pupil, the amplitude spread function is given by [13]:

$$h(x,y)=\frac{2\pi w^2}{(\bar{\lambda}z)^2}\frac{J_1(2\pi w\sqrt{x^2+y^2}/\bar{\lambda}z)}{2\pi w\sqrt{x^2+y^2}/\bar{\lambda}z}, \tag{7}$$

where $J_1(\cdots)$ is first order the Bessel Function of the first kind, w is the radius of the exit pupil, z is the image distance, and $\bar{\lambda}$ is the mean wavelength of the emitting quasi-monochromatic light. After substitution of Eq. (7) into Eq. (6), the distributions of the Stokes parameters for polarization imaging can be estimated for various separations of the two points and various degrees of polarization coherence.

3. SPATIAL RESOLUTION FOR POLARIZATION IMAGING

To give a complete investigation of polarization imaging with all the Stokes parameters and to highlight the unique characteristics of the polarization imaging with S_l $(l=1,2,3)$, we will start our analysis from polarization image of S_0, which shares the similar characteristics of scalar optics case. Fig. 1 shows the distribution of the Stokes parameter S_0 in polarization image of two points for various values of $\tilde{\mu}_{S_0}$ which has be chosen from -1.0 to 1.0 in steps of 0.2. In this example, the separation has been chosen as $3.2\bar{\lambda}f/(\pi w)$ and the locations of the image for the two points given by geometrical optics. Similar to the well-known two-point resolution with partially coherent light [6,7], the separation judged from the maxima of the resultant S_0 distribution is not always equivalent to the true geometrical separation. Note that when $\tilde{\mu}_{s_0} = -1.0$, the two points are illuminated coherently but with a π phase difference, and the distribution of S_0 at the mid-point between them always falls to zero [14]. The use of $\tilde{\mu}_{s_0} = -1.0$ for improvement of two points' resolution had already been made in lithography with the name of phase-shift mask technique [15]. With an increase of $\tilde{\mu}_{s_0}$, the distribution for the Stokes parameter S_0 become shallower at its midpoint. When $\tilde{\mu}_{s_0} = 0$, the curve for the resultant S_0 is almost flat without a dip, indicating that these two point sources become unresolvable in the polarization image for S_0.

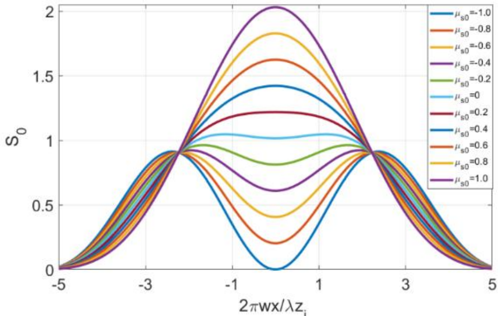


Fig.1. Distribution of S_0 in polarization image of two points for various values of $\tilde{\mu}_{S_0}$.

Note the fact that the only measurable quality in a diffraction-limited polarization imaging system is the separation of the two peaks in the resultant distribution of polarization image for S_0. Following Crimes and Thompson [6,7], we use a dimensionless parameter R_{S_0} to present the two-point resolution for polarization imaging. Here, R_{S_0} has been given as the ratio of the measurable separation of the two peaks of the resultant Stokes parameter S_0 to the separation of the two image points formed by geometrical optics. For a perfect polarization imaging system without diffraction, clearly, R_{S_0} should become unity and any other values different from one indicate a distortion in polarization imaging. Fig. 2 shows the ratio R_{S_0} as a function of the ideal separation of two image points for various value of $\tilde{\mu}_{S_0}$ where all these curves are seen to oscillate around the value $R_{S_0} = 1$. As expected for a coherent illumination for two points with a π phase difference with $\tilde{\mu}_{s_0} = -1.0$, the ratio R_{S_0} never becomes zero regardless of their separation, indicating that these two points are always resolvable in polarization imaging of S_0. For the rest value of $\tilde{\mu}_{s_0}$, these two points becomes unresolvable with $R_{S_0} = 0$ in the diffraction limited polarization imaging system when their separation becomes small. It's interesting to note that a better resolution can be achieved for the polarization image of S_0 with a decreased value of the degree of the generalized Stokes parameter $\tilde{\mu}_{s_0}$.

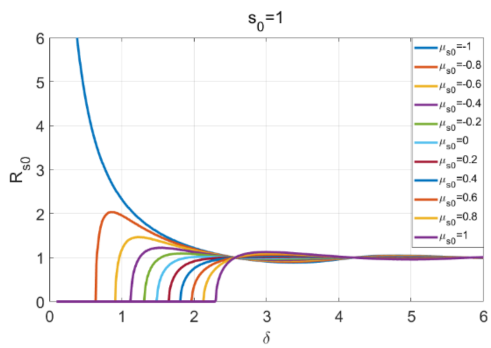


Fig.2. The ratio R_{S_0} asa function of the true geometrical separation of two image points for various values of $\tilde{\mu}_{S_0}$.

Note from Eqs. (6) that both the coherence properties $\tilde{\mu}_{S_l}$ and polarization properties s_l of the illumination will play critical role to the polarization image formation of S_l $(l = 1,2,3)$. For a given birefringence-free imaging system with a fixed aperture, the effect of varying $\tilde{\mu}_{S_l}$ and s_i of illumination and a various values of the point separation to polarization imaging with S_l $(l = 1,2,3)$ can be investigated in a similar way. Figure 3 plots the distribution of the Stokes parameters S_l $(l = 1,2,3)$ in polarization image of two points under illumination of partially coherent and partially polarized light with various values of $\tilde{\mu}_{S_i}$.Unlike the Stokes parameters S_0 in polarization image with its value being always non-negative, the rest of Stokes parameters S_l $(l = 1,2,3)$ will be not necessarily positive. When the Stokes parameters in polarization image become negative, the separation for image resolvability assessment will be judged from the two negative-valued valleys as the polarization images for these two closely spaced points. Similar to the case for the Stokes parameter S_0, the separation from the resultant S_l $(l = 1,2,3)$ is not always equivalent to the true geometric separation marked by the dashed lines. For a certain normalized Stokes parameter s_i, the distribution of the Stokes parameter S_l in polarization image of two points varies under different values of $\tilde{\mu}_{S_i}$ for illumination light. It's interesting to see from Fig.

3 that the distribution of S_l is flipping upside down if the illumination condition for partially coherent and partially polarized light is flipping for $\tilde{\mu}_{S_l}$.

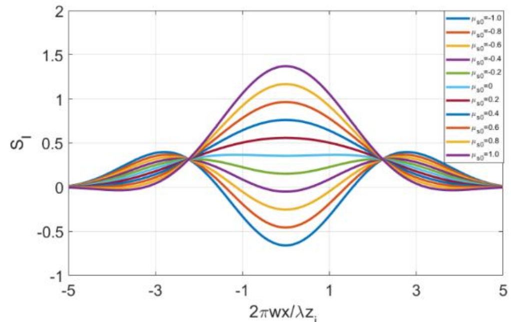
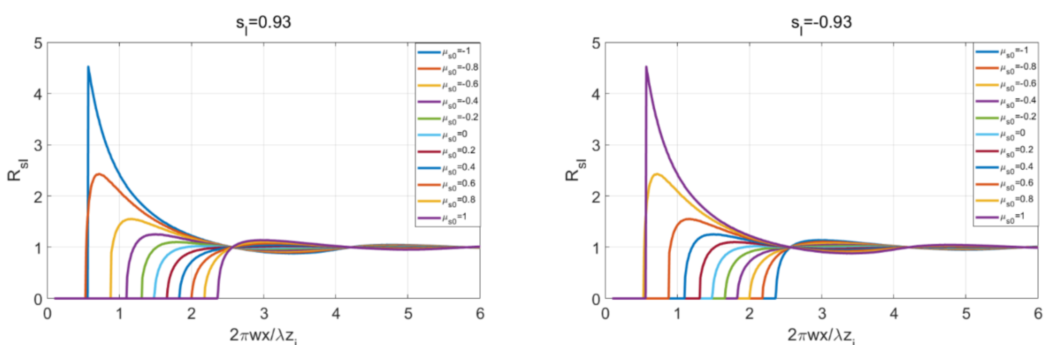


Fig. 3 Distribution of $S_l(l=1,2,3)$ in polarization image of two points for various values of $\tilde{\mu}_{S_l}$.

Similar to the case for S_0 , three dimensionless parameters $R_{S_l}(l=1,2,3)$ defined as the ratio of the measurable separation of the two peaks/valleys of the resultant Stokes S_l to the ideal separation of the two image points given by geometrical optics. Figure 4 shows the ratio R_{S_l} as a function of the ideal separation of the two points in polarization image from various values of $\tilde{\mu}_{S_l}$. As expected for large separation of the two points, all these curves are seen to oscillate around the values $R_{S_l}=1$. When flipping the illumination condition from $(\tilde{\mu}_{S_l}, s_l)$ to $(-\tilde{\mu}_{S_l}, -s_l)$, the two curves for R_{S_l} are identical indicating the same spatial resolution for the Stokes image of S_l . In order to enhance the spatial resolution for a diffraction limited polarization imaging system, we can control the illumination condition by a careful selection of its coherence and polarization properties of a partially polarized and partially coherent light source. Note from the fact that the proposed three degrees of the generalized Stokes parameters $\tilde{\mu}_{S_l}$ are independent to each other. Those are also true for the three normalized Stokes parameters s_l . Therefore, the same two closely spaced points resolvable for a certain Stokes parameter in the same polarization imaging system may become unresolvable for other Stokes parameter.



(a) (b)

Fig. 4. The ratio $R_{S_l}(l=1,2,3)$ .When s_1 is 0.93(a) (a) and -0.93(b), respectively, the illuminance distribution of the polarized image changes with a change from $\tilde{\mu}_{s1}=-1.0$ to 1.0.

We note in closing of our discussion that the question of when two closely spaced point sources for polarization image are resolved is a complex one, and the spatial resolution of a polarization imaging system is very much dependent upon the coherence and polarization properties of the illumination light source. In fact, the ability to resolve these two points depends fundamentally on the signal-to-noise ratio associated with the imaging system. Therefore, any criteria without taking account of noise are subjective. Nonetheless, such discussion in this letter may yield some useful rules of thumb for engineering practice, especially for the polarization-related optical engineering. Further, the proposed degree of polarization coherence will make possible the characterization of the polarization and coherence properties of stochastic electromagnetic radiation, and introduces new opportunities to explore other statistical optical phenomena.

4. CONCLUSIONS

In summary, we reviewed our recent work on the unified theory of polarization and coherencein stochastic electromagnetic beams. Based on the wave properties associated with the mutual coherence tensor, we introduced several new concepts associated with the coherence tensor wave and provided the definition for the degree of coherence tensor. We provided a novel optical system for the full-field visualization of coherence tensor and proposed an unconventional holography: coherence tensor holography to synthesize arbitrarily the coherence for stochastic electric fields. The degree of generalized Stokes parameters has been proposed and the two-point resolution issue has been revisited for polarimetric imaging.

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